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BOUNDARY INTEGRALS FOR THE WAVE EQUATION IN THE PROBLEM OF PLANE WAVE DIFFRACTION

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Construction of boundary integrals for the wave equation, based on the analysis of eigenfunctions used in solving self-similar problems of plane wave diffraction, is presented for plane and three-dimensional cases. Derivation of these integrals is carried out similarly to the derivation of the Poisson's integral for the Laplace equation. The result obtained in the plane case can be extended to the problem of diffraction of a wide class of plane waves on a wedge. Extension to the three-dimensional case is obtained for problems similar to those of a plane wave diffraction on a thin delta wing moving at supersonic speed. An example of the construction in quadratures of the problem of diffraction of a single wave on a thin delta wing travelling at a speed higher than the speed of sound is presented. The present investigation is to a considerable extend based on ideas in [1 — 6].

1. We consider the wave equation

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}-\frac{\partial^2\Phi}{\partial t^2}=0 \tag{1.1}$$

For the diffraction of a plane wave of the form

$$\Phi_n=(t-r\cos(\alpha+\theta))^n \quad (\theta=\operatorname{arctg}(y/x),\ r=\sqrt{x^2+y^2})$$

on a wedge the solution has, also, the form of a homogeneous function of dimension n with respect to t and r. The relation between homogeneous solutions of zero and n dimension is defined by formula (1.2) in [7].

The homogeneous function of zero dimension satisfies the Laplace equation

$$\frac{\partial^2\Phi_0}{\partial R^2}+\frac{1}{R}\frac{\partial\Phi_0}{\partial R}+\frac{1}{R^2}\frac{\partial^2\Phi_0}{\partial\theta^2}=0,\quad R=\frac{t}{r}-\sqrt{\frac{t_2}{r^2}-1}$$

for which the Poisson's integral

$$\Phi_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{f(\psi)(1-R^2)\,d\psi}{1+R^2-2R\cos(\psi-\theta)},\quad \Phi_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{f(\psi)\sqrt{t^2-r^2}\,d\psi}{t-r\cos(\psi-\theta)}$$

is known.

Using formula (1.2) from [7], for the homogeneous solution of dimension n we obtain the boundary integral

$$\Phi_n=\frac{2^n(n!)^2}{2\pi(2n!)}\int_0^{2\pi}\frac{f_n(\psi)(t^2-r^2)^{n+1/2}\,d\psi}{[t-r\cos(\psi-\theta)]^{n+1}},\quad f_n(\theta)=\frac{\Phi_n}{t^n}\bigg|_{t=r}$$

For any arbitrary plane wave and boundary conditions

$$\Phi|_{r=t}=f(0,t)=\sum_{n=0}^{\infty}f_n(\theta)t^n$$

at the circle $r = t$ we have inside the circle $r \leqslant t$

$$\Phi = \frac{1}{2\pi} \int_0^{2\pi} \frac{\sqrt{t^2 - r^2}}{t - r\cos(\psi - \theta)} \left\{ \int_0^1 f(\psi, \eta)\, d\lambda + 2 \int_0^1 \eta f_n'(\psi, \eta)\, d\lambda \right\} d\psi \qquad (1.2)$$

$$\eta = 2 \frac{t^2 - r^2}{t - r\cos(\psi - \theta)} \lambda (1 - \lambda)$$

The relationship

$$\int_0^1 [\lambda(1 - \lambda)]^n\, d\lambda = \frac{(n!)^2}{(2n+1)!}$$

was used in the derivation of the integral (1.2).

Integral (1.2) is valid if Φ is of period T which is equal 2π with respect to θ. This occurs, for example, in the analysis of diffraction of a plane wave on a plate moving at supersonic speed. When $T = 2\pi k$ ($k \neq 1$ and is an integer), the result is more complex. In that case the Poisson's integral is of the form

$$K_0(t, r, \theta, \psi) = \frac{1 - R_1^2}{1 + R_1^2 - 2R_1 \cos(\psi_1 - \theta_1)} \qquad (1.3)$$

$$R_1 = R^{1/k} = \left(\frac{t}{r} - \sqrt{\left(\frac{t}{r}\right)^2 - 1} \right), \quad \theta_1 = \frac{\theta}{k}, \quad \psi_1 = \frac{\psi}{k}$$

Taking into account (1.3) we obtain a boundary integral of the form

$$\Phi = \frac{\sqrt{t^2 - r^2}}{4\pi^2 i} \int_0^{2\pi} d\psi_1 \oint \frac{F(\psi_1, \xi) K_0(t - 1/\zeta, r, \theta, \psi)\, d\zeta}{\zeta \sqrt{(t - 1/\zeta)^2 - r^2}}$$

$$F(\psi_1, \xi) = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} f_n(\psi_1) \xi^n = \int_0^1 f(\psi_1, \eta)\, d\lambda + 2 \int_0^1 \eta f_n'(\psi_1, \eta)\, d\lambda$$

$$i = \sqrt{-1}, \quad \xi = 2(t^2 - r^2)\zeta, \ \eta = \xi\lambda(1 - \lambda), \ \Phi|_{r=t} = f(\theta_1, t)$$

where ζ is a complex quantity. The integral over the closed contour is taken over a curve which is little different from $\zeta = 0$ and surrounds point $\zeta = 0$.

2. Let us consider homogeneous solutions of Eq. (1.1) of half-integer dimension. Let $\Phi_\beta = r^\beta \varphi_\beta (t/r, \theta)$, then φ_β satisfies the equation

$$(w^2 - 1) \frac{\partial^2 \varphi_\beta}{\partial w^2} - (2\beta - 1) \frac{\partial \varphi_\beta}{\partial w} + \beta^2 \varphi_\beta + \frac{\partial^2 \varphi_\beta}{\partial \theta^2} = 0, \quad w = \frac{t}{r} \qquad (2.1)$$

As before, we consider solutions of Eq. (2.1) for specified conditions at the circle $w = 1$ ($r = t$). To derive such solutions we examine the eigenfunctions of Eq. (2.1) at the separation of variables w and θ. The eigenfunctions that depend on θ are equal $\cos\mu\theta$ and $\sin\mu\theta$. For the eigenfunctions that depend on w we then obtain the second order ordinary differential equation

$$(w^2 - 1) y_\beta'' - (2\beta - 1) w y_\beta' + (\beta^2 - \mu^2) y_\beta = 0 \qquad (2.2)$$

Equation (2.2) has two important properties

$$y_{\beta-1} = y_\beta', \quad y_\beta = w y_{\beta-1} - \frac{(w^2 - 1)}{2\beta - 1} y_{\beta-1}' \qquad (2.3)$$

of which the last loses its meaning when $\beta = 1/2$. If $\beta = -1/2$ and $\mu = \nu + 1/2$, (2.2) becomes the known Legendre equation.

Using the first of formulas (2. 3), we obtain the solution of (2. 2) for $\beta = 1/_2$

$$y_{1/_2} = (w^2 - 1)\left(C_1 \frac{dP_\nu(w)}{dw} + C_2 \frac{dQ_\nu(w)}{dw}\right)$$

where $(P_\nu(w)$ and $Q\ (w)$ are Legendre functions of the first and second kind, respectively.

Since the solution is being determined inside the circle $r \leqslant t$, we specify that $y_{1/_2} = 1$ for $w = 1$, and that the quantity $w^{-1/_2} y_{1/_2}$ must be finite when $w \to \infty$. Then

$$y_{1/_2} = -(w^2 - 1)\frac{dQ_\nu(w)}{dw} \tag{2.4}$$

The logarithmic singularity of $Q_\nu(w)$ at $w = 1$ is eliminated from (2. 4) by the factor $(w^2 - 1)$. For example, for $\nu = -1/_2$

$$y_{1/_2} = \sqrt{\frac{w+1}{2}}\, E\left(\sqrt{\frac{2}{w+1}}\right)$$

where E is a total elliptic integral of the second kind. Thus the solution of Eq. (1. 1) with $\beta = 1/_2$ is in the general case of the form

$$\Phi_{1/_2} = -\sqrt{r}\,(w^2 - 1)\frac{\partial}{\partial w}\left\{\frac{A_0}{2} Q_{-1/_2}(w) + \sum_{(\mu)} (A_\mu \cos\mu\theta + B_\mu \sin\mu\theta)\, Q_{\mu-1/_2}(w)\right\}$$

If the period of $\Phi_{1/_2}$ with respect to θ is 2π, then $\mu = m$, where m is an integer. To derive the boundary integral we use the Laplace integral representation for Legendre functions [8] and the expressions for the coefficients of the Fourier series expansion

$$Q_\nu(w) = \int_0^\infty \frac{d\lambda}{(w + \sqrt{w^2 - 1}\,\mathrm{ch}\,\lambda)^{\nu+1}}$$

$$A_m = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\cos m\psi\, d\psi, \qquad B_m = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\sin m\psi\, d\psi$$

$$f(\theta) = r^{-1/_2}\Phi_{1/_2}\Big|_{r=t}$$

Then

$$\Phi_{1/_2} = -\frac{\sqrt{r}\,(w^2 - 1)}{2\pi}\frac{\partial I}{\partial w}, \quad I = \int_0^{2\pi}\int_0^\infty \frac{f(\psi)\, d\psi\, d\lambda\, (R_\lambda^2 - 1)}{\sqrt{R_\lambda}\,(1 + R_\lambda^2 - 2R_\lambda\cos(\psi - \theta))} \tag{2.5}$$

$$R_\lambda = w + \sqrt{w^2 - 1}\,\mathrm{ch}\,\lambda$$

Carrying out integration in formula (2. 5) with respect to λ, we obtain

$$\Phi_{1/_2} = \frac{(t^2 - r^2)}{4\sqrt{2}}\int_0^{2\pi} \frac{f(\psi)\, d\psi}{(t - r\cos(\psi - \theta))^{3/_2}}$$

If the period of function $\Phi_{1/_2}$ with respect to θ is equal $2\pi k$, it is necessary to set $\mu = m / k$, where k and m are integers. Then

$$\Phi_{1/_2} = -\frac{\sqrt{r}\,(w^2 - 1)}{2\pi}\frac{\partial I_1}{\partial w}, \quad I_1 = \int_0^{2\pi}\int_0^\infty \frac{f(\psi_1)\, d\psi_1\, d\lambda\, (\rho^2 - 1)}{\sqrt{R_\lambda}\,(1 + \rho^2 - 2\rho\cos(\psi_1 - \theta_1))} \tag{2.6}$$

$$\theta_1 = \theta / k, \quad \rho = (R_\lambda)^{1/k}$$

For convenience of computation the integral I_1 in formula (2. 6) can be represented for $k = 2$ as

$$I_1 = \int_0^{2\pi} 2\sqrt{R} f(\psi_1)\, d\psi_1 \int_0^1 \frac{\xi\, d\xi\, (1-\xi R)}{\sqrt{(1-\xi^2)(1-R^2\xi^2)}\,(1+\xi^2 R - 2\xi\sqrt{R}\cos(\psi_1-\theta_1))}$$

$$R = w - \sqrt{W^2-1}$$

It will be seen that the integrand of I_1 is expressed in terms of complete elliptic integrals.

Using the second of formulas (2. 3), we obtain

$$y_{n+1/2} = \frac{(-1)^n (w^2-1)^{n+1}}{2^n n!} \frac{d^n}{dw^n}\left(\frac{y_{1/2}}{w^2-1}\right)$$

which makes it possible to carry out the extension to any plane wave, as is done in Sect.1.

3. Let us consider the wave equation in the space

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} - \frac{\partial^2\Phi}{\partial t^2} = 0 \qquad (3.\ 1)$$

Borovikov had shown in [6] that solutions of Eq.(3. 1), which are represented by functions of dimension $-1/2$ homogeneous with respect to t and $q = \sqrt{x^2+y^2+z^2}$, reduce to the Dirichlet problem for the Laplace equation. The results obtained here make it possible to extend that conclusion to homogeneous solutions of Eq.(3. 1) of any half-integer dimension (*).

If $\Phi_{n-1/2}$ is a solution homogeneous with respect to t and q of Eq.(3. 1) of dimension $n - 1/2$, $\Phi_{-1/2}$ is a homogeneous solution of the similar equation of dimension $-1/2$ and $t^{-n+1/2}\Phi_{n-1/2}|_{q=t} = t^{1/2}\Phi_{-1/2}|_{q=t}$, then

$$\Phi_{n-1/2} = \frac{(-1)^n 2^n n!(t^2-q^2)^{n+1/2}}{(2n)!} \frac{\partial^n}{\partial t^n}\left(\frac{\Phi_{-1/2}}{\sqrt{t^2-q^2}}\right) \qquad (3.\ 2)$$

For solutions homogeneous with respect to t and q of Eq.(3. 1) of integral dimension we obtain the similar formula

$$\Phi_n = \frac{(-1)^n (t^2-q^2)^{n+1}}{2^n n!} \frac{\partial^n}{\partial t^n}\left(\frac{\Phi_0}{t^2-q^2}\right) \qquad (3.\ 3)$$

where Φ_0 and Φ_n are solutions homogeneous with respect to t and q of Eq.(3. 1) of dimension 0 and n, respectively.

Taking into account results obtained by Borovikov [6], for the homogeneous solution of $-1/2$ dimension of Eq.(3. 1) we obtain the boundary integral of the form

$$\Phi_{-1/2} = \frac{1}{4\pi\sqrt{2}} \int_0^{\pi}\int_0^{2\pi} \frac{F(\omega_1, \theta_1)\sin\omega_1\, d\omega_1\, d\theta_1 \sqrt{t^2-q^2}}{(t - q\cos\gamma)^{3/2}}$$

$$\cos\gamma = \cos\omega\cos\omega_1 + \sin\omega\sin\omega_1\cos(\theta_1-\theta), \quad F(\omega, \theta) = \sqrt{t}\,\Phi_{-1/2}|_{t=q}$$

where ω and θ are angular spherical coordinates. Taking into consideration (3. 2) we have

*) V. V. Tret'iakov, On the problem of reducing in the self-similar case the solution of the wave equation in space to the solution of the Laplace equation. Theses of Proceedings of the fourth All-Union Symposium on the Propagation of Elastic and Elastico-Plastic Waves, Kishinev, 1968.

$$\Phi_{n-1/2} = \frac{2n+1}{4\pi\cdot 2^{n+1/2}} \int_0^{\pi}\int_0^{2\pi} \frac{F_{n-1/2}(\omega_1, \theta_1)\sin\omega_1\,d\omega_1\,d\theta_1\,(t^2-q^2)^{n+1/2}}{(t-q\cos\gamma)^{n+3/2}}$$

For the homogeneous solution of dimension zero we have the expansion in eigenfunctions

$$\Phi_0 = -\sum_{m=0}^{\infty}(w^2-1)\,Y_m(\omega, \theta)\frac{dQ_m(w)}{dw}, \quad w = \frac{t}{q}$$

where $Q_m(w)$ is the Legendre function of the second kind of integral dimension and $Y_m(\omega, \theta)$ is a spherical harmonic of integral order.

Using the Laplace integral representation for Legendre functions and also the Laplace formula (see e. g., [9])

$$Y_m(\omega, \theta) = \frac{2m+1}{4\pi}\int_0^{\pi}\int_0^{2\pi} F(\omega_1, \theta_1)\,P_m(\cos\gamma)\sin\omega_1\,d\omega_1\,d\theta_1$$

where $P_m(\cos\gamma)$ is a Legendre's polynomial of power m, $F(\omega, \theta) = \Phi_0\,|_{t=q}$, and the equality

$$\sum_{m=0}^{\infty}\frac{(2m+1)\,P_m(\cos\gamma)}{R_\lambda^{m+1}} = \frac{R_\lambda{}^2-1}{(1+R_\lambda{}^2-2R_\lambda\cos\gamma)^{3/2}}$$

after summation and integration with respect to λ, we obtain the following boundary integral:

$$\Phi_0 = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\frac{F(\omega_1, \theta_1)\sin\omega_1\,d\omega_1\,d\theta_1\,(t^2-q^2)}{[t-q\cos\gamma]^2} \tag{3.4}$$

and taking into account (3.3)

$$\Phi_n = \frac{(n+1)}{4\pi\cdot 2^n}\int_0^{\pi}\int_0^{2\pi}\frac{F(\omega_1, \theta_1)\sin\omega_1\,d\omega_1\,d\theta_1\,(t^2-q^2)^{n+1}}{(t-q\cos\gamma)^{n+2}} \tag{3.5}$$

If $q = t$ is specified at the sphere surface

$$\Phi\,|_{q=t} = f(\omega, \theta, t) = \sum_{n=0}^{\infty} f_n(\omega, \theta)\,t^n$$

then, taking into account the integral (3. 5), we obtain

$$\Phi = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\frac{\sin\omega_1\,d\omega_1\,d\theta_1\,(t^2-q^2)}{(t-q\cos\gamma)^2}\frac{d}{d\xi}(\xi f(\omega_1, \theta_1, \xi))$$

$$\xi = \frac{1}{2}\frac{t^2-q^2}{t-q\cos\gamma}$$

In the case of axial symmetry the boundary integral is obtained by setting in formula (3.4) $F(\omega_1, \theta_1) = F(\omega_1)$ and integrating with respect to θ_1. That integral is of the form

$$\Phi_0 = \frac{1}{2}\int_0^{\pi}\frac{F(\omega_1)\sin\omega_1\,d\omega_1\,(t-q\cos\omega\cos\omega_1)(t^2-q^2)}{[(t-q\cos\omega\cos\omega_1)^2-q^2\sin^2\omega\sin^2\omega_1]^{3/2}}$$

It should be noted that the obtained here results are only valid when the period of the potential with respect to θ, is equal 2π. In comparison with the plane problem this corresponds to the case when it is not necessary to carry out conformal representation.

4. As an example of the obtained results we present the solution of the problem of diffraction of a single wave on a delta wing moving at constant supersonic speed.

Let a delta wing whose angle of sweep at the vertex is $\pi/2 - \nu$ move along its axis of symmetry in the negative direction of the z-axis of a Cartesian system of coordinates at constant supersonic velocity $(M > 1)$. The y-axis is normal to the wing. The angular spherical coordinates are chosen so that

$$\theta = \operatorname{arctg}\frac{y}{x}, \quad \omega = \operatorname{arctg}\frac{\sqrt{x^2+y^2}}{z}$$

Let a single plane wave

$$\Phi = H(t - q\cos\omega\cos\alpha + q\sin\omega\sin\theta\sin\alpha)$$
$$q = \sqrt{x^2+y^2+z^2}, \quad \alpha = \text{const}$$

where H is a unit function, impinge on the wing.

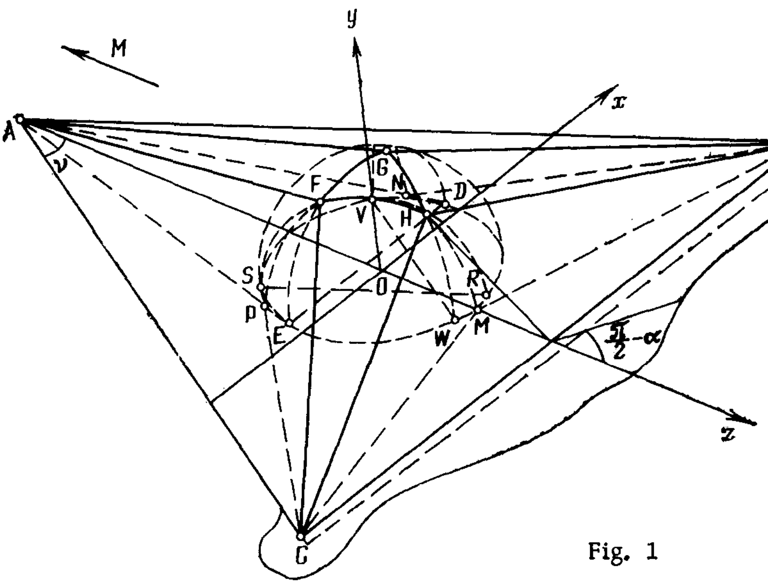

Fig. 1

The pattern of diffraction is schematically shown in Fig. 1. The flow is three-dimensional only inside the diffracted hemisphere centered at point O, and in remaining regions the solution can be obtained by using equations which define plane motion.

Thus, for example, in regions $ABNG$ and $ACPF$ the solution is a constant of the form

$$\Phi = 1 + L, \quad L = \frac{M \operatorname{tg}\nu \sin\alpha}{\sqrt{(M^2-1)\operatorname{tg}^2\nu - 1}}$$

In region $CBHM$ the quantity Φ is also constant and equal two.

Inside the half-cones with vertices at points A, B and C the solution is obtained by reducing the three-dimensional wave equation to a two-dimensional one by the substitution for the variables z_1 and t of the single one $\tau = (\zeta_0 t - z_1) / \sqrt{\zeta_0^2 - 1}$, where ζ_0 is a constant that defines the velocity of motion of the half-cone vertex along its axis.

That substitution reduces equation

$$\frac{\partial^2\Phi}{\partial x_1^2} + \frac{\partial^2\Phi}{\partial y_1^2} + \frac{\partial^2\Phi}{\partial z_1^2} - \frac{\partial^2\Phi}{\partial t^2} = 0$$

to the form

$$\frac{\partial^2\Phi}{\partial x_1^2}+\frac{\partial^2\Phi}{\partial y_1^2}-\frac{\partial^2\Phi}{\partial\tau^2}=0$$

The admissibility of such substitution becomes clear, if we introduce a moving system of coordinates with its origin at the half-cone vertex. The pattern of flow is then stationary and it is seen that it is independent of the fourth variable.

As regards the half-cones with vertices at points B and C the coordinates x_1, y_1, z_1, and the quantity ζ_0 are defined thus:

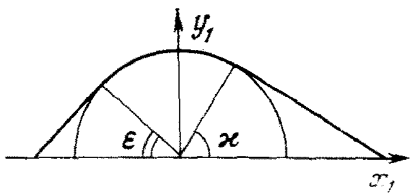

Fig. 2

$$\zeta_0 = 1 / (\cos\alpha\cos\beta),\quad y_1 = y$$
$$z_1 = z\cos\beta \pm x\sin\beta,\ x_1 = x\cos\beta \mp z\sin\beta$$
$$\cos\beta = [1+(M\cos\alpha+1)^2\,\mathrm{tg}^2\,\nu]^{-1/2}$$

where the upper and lower signs relate, respectively, to vertices B and C.

For the half-cone with the vertex at point A we have $\zeta_0 = M$, $y_1 = y$, $z_1 = -z$ and $x_1 = x$.

It remains to determine in planes perpendicular to the half-cone axes the angles which separate different conditions on the surface of half-cones (see Fig. 2). We have

$$\sin\varkappa = \frac{\sin\alpha\cos\delta}{L\cos\beta},\quad \sin\varepsilon = \frac{\sin\delta}{\cos\beta}\ (B)$$
$$\sin\varkappa = \sin\delta / \cos\beta,\quad \sin\varepsilon = \sin\alpha\cos\delta / (L\cos\beta)\quad (C)$$
$$\sin\varepsilon = \sin\varkappa = M\sin\alpha / (L\sqrt{M^2-1})\quad (A)$$
$$\cos\delta = [(M\cos\alpha+1)^2\,\mathrm{tg}^2\,\nu + \sin^2\alpha]^{-1/2}$$

where the letters in parentheses denote the vertices of related half-cones.

For the boundary conditions at the surface of half-cones we have

$$0\leqslant\theta_1<\varkappa:\Phi=1+L;\quad \varkappa<\theta_1<\pi-\varepsilon:\Phi=1;\quad \pi-\varepsilon<\theta_1\leqslant\pi:\Phi=2\quad (B)$$

$$0\leqslant\theta_1<\varkappa:\Phi=2,\quad \varkappa<\theta_1<\pi-\varepsilon:\Phi=1,\quad \pi-\varepsilon<\theta_1\leqslant\pi:\Phi=1+L\ (C)$$

$$0\leqslant\theta_1<\varkappa,\ \pi-\varkappa<\theta_1\leqslant\pi:\Phi=1+L;\ \varkappa<\theta_1<\pi-\varkappa:\Phi=1\ (A)$$

At the wing surface we have for all half-cones $\partial\Phi/\partial y=0$. This makes it possible to continue symmetrically the boundary conditions at the half-cone surfaces to the whole surface of cones and apply the Poisson's integral. Since boundary conditions at the surfaces of cones are expressed in terms of sets of constant quantities, it is sufficient to adduce the solution for one of the cones (e.g., for the cone with the vertex point B); solutions for the remaining cones are derived similarly. This solution is of the form

$$\Phi = 1+\frac{L}{\pi}\left[\mathrm{arctg}\left(\frac{1+R_1}{1-R_1}\,\mathrm{tg}\,\frac{\varkappa-\theta_1}{2}\right)+\mathrm{arctg}\left(\frac{1+R_1}{1-R_1}\,\mathrm{tg}\,\frac{\varkappa+\theta_1}{2}\right)\right]+$$
$$\frac{1}{\pi}\left[\mathrm{arctg}\left(\frac{1+R_1}{1-R_1}\,\mathrm{tg}\,\frac{\pi+\varepsilon-\theta_1}{2}\right)-\mathrm{arctg}\left(\frac{1+R_1}{1-R_1}\,\mathrm{tg}\,\frac{\pi-\varepsilon-\theta_1}{2}\right)\right]$$
$$R_1 = \frac{\tau}{\sqrt{x_1^2+y_1^2}}-\sqrt{\frac{\tau^2}{x_1^2+y_1^2}-1},\quad \theta_1 = \mathrm{arctg}\,\frac{y_1}{x_1}$$

To simplify further exposition we denote the solutions inside cones with vertices at points A, B and C, respectively, by

$$\Phi = 1 + \Phi_1, \quad \Phi = 1 + \Phi_2, \quad \Phi = 1 + \Phi_3$$

Note that there are three regions adjacent to the hemisphere in which the half-cones intersect. In these regions the solution is not a simple sum of solutions for two half-cones (if a continuous variation of solution at the boundary of these is required).

For example, the solution in region $VNDG$ is to be defined as

$$\Phi = 1 + \Phi_1 + \Phi_2 - L$$

since this solution satisfies boundary conditions and the wave equation. Similarly, in region $FSPE$ the solution is

$$\Phi = 1 + \Phi_1 + \Phi_3 - L$$

and in region $HRMW$

$$\Phi = 1 + \Phi_2 + \Phi_3 - 1 = \Phi_2 + \Phi_3$$

After the solutions have been determined in all regions adjacent to the sphere, the solution inside the latter is obtained with the use of formula (3.4). Because of the condition that at the wing surface $\partial\Phi/\partial y = 0$, the boundary conditions at the sphere surface must be symmetric about the surface $y = 0$.

The derived solution defines the flow for $y > 0$. The solution for the wing underside can be obtained by using the equality $\Phi_- = -\Phi_+ + 2$, where Φ_+ is the solution for $y > 0$, and Φ_- the solution for $y < 0$.

It should be noted that the results presented here are valid only when the base of the diffracted sphere does not extend beyond the wing leading edges. Otherwise it is necessary to carry out special investigations different from those described here.

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